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TOTAL CORDIAL MAGICNESS IN TRIPLICATION OF GRAPHS

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ABSTRACT

In this paper, we investigate the existence of total cordial edge magic labeling and total cordial vertex magic labeling for the extended triplicate graph of twig by presenting algorithms.

INTRODUCTION

The origin of the study of graph labeling as a major area of graph theory can be traced to a research paper by Rosa [6]. A labeling of a graph G is a mapping that carries a set of graph elements, usually the vertices and/or edges, into a set of numbers, usually integers, called labels. Various labeling schemes have been introduced so far and explored as well by many researchers. Many kinds of labelings have been studied in the literature and an excellent survey of graph labelings can be found in [4].

Cahit [3] introduced the notion of cordial labeling. A function f from V to {0,1} such that each edge uv receives the label |f (u) – f (v)| is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one. A function *f* from V to {0,1} such that each edge uv receives the label |f (u) – f (v)| is said to be total cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

Kotzig & Rosa [5] defined a magic valuation of a graph G (V,E) as a bijection f from V ∪ E to {1, 2,... , |V ∪ E|} such that for all edges xy, f(x) + f(y) + f(xy) is constant (called the magic constant). This notion was rediscovered by Ringel and Llad '0' in 1996 who called this labeling edge-magic [8].

MacDougall, Miller, Slamin, & Wallis [7] introduced the notion of a vertex-magic total labeling in 1999. For a graph G (V,E) an injective mapping *f* from V∪E to the set {1, 2,... ,

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|V|+|E|} is a vertex-magic total labeling if there is a constant k, called the magic constant, such that for every vertex v, f(v) + Σf(vu) = k where the sum is over all vertices u adjacent to v.

Bala & Thirusangu [1] introduced the notion of extended triplicate graph of a path. Let V = {$v_1, v_2, \ldots, v_{n+1}$} and E = { $e_1, e_2, \ldots, e_n$} be the vertex and Edge set of a path P_n. For every $v_i \in V$, construct an ordered triple {v_i , v_i' ,v_i''} where, $1 \leq i \leq n+1$ and for every edge $v_iv_j \in E$, construct four edges v_iv_j', $v_j' v_i''$, v_jv_i' and $v_i' v_j''$ where, j = i +1, then the graph with this vertex set and edge set is called a Triplicate Graph of a path P_n. It is denoted by TG (P_n). Clearly the Triplicate graph TG (P_n) is disconnected. Let V_1 = { $v_1, v_2, \ldots, v_{3n+1}$} and E_1 = { $e_1, e_2, \ldots, e_{4n}$} be the vertex and edge set of TG (P_n). If n is odd, include a new edge (v_{n+1} , v_1) and if n is even, include a new edge (v_n ,v_1) in the edge set of TG (P_n). This graph is called the Extended Triplicate of the path P_n and it is denoted by ETG (P_n).

Motivated by the study, in the present work, we investigate the existence of total cordial edge magic labeling and total cordial vertex magic labelings for the extended triplicate graph of a twig.

PRELIMINARIES

In this section we provide basic definitions and other information which are prerequisites for the present investigations.

2.1 Twig Graph

A graph *G (V,E)* obtained from a path by attaching exactly two pendent edges to each internal vertices of the path is called a Twig graph. A twig obtained from a path with n-internal vertices is denoted as T_n. If n is even then $T_n$is called an even twig. T_n is said to be odd if n is odd. Generally, a twig T_n has 3n+1 edges and 3n+2 vertices.

2.2 Structure of the Extended Triplicate Graph of Twig

In this section, we provide the structure of the extended triplicate graph of twig by presenting algorithm.

Algorithm 2.2:
Input: Twig graph
Procedure triplicate of twig graph T_n
for i = 1 to n+2 **do**
    $V_1 \leftarrow \{v_i \cup v_i' \cup v_i''\}$
end for

for i = 2 to n+1 **do**
    $V_2 \leftarrow \{u_i \cup u_i' \cup u_i''\}$ u_i; $V_3 \leftarrow \{w_i \cup w_i' \cup w_i''\}$
end for
    $V \leftarrow V_1 \cup V_2 \cup V_3$

for i = 1 to n+1 **do**
    $E_1 \leftarrow (v_iv_{i+1}') \cup (v_i'v_{i+1}) \cup (v_i''v_{i+1}') \cup (v_i'v_{i+1}'')$

end for

for i = 2 to n+1 **do**

$E_2 \leftarrow (v_i' u_i) \cup (v_i u_i') \cup (v_i' w_i) \cup (v_i w_i') \cup (v_i' u_i'') \cup (v_i'' u_i') \cup (v_i' w_i'') \cup (v_i'' w_i')$

end for

$E \leftarrow E_1 \cup E_2$

end procedure

Output: Triplicate graph of twig T_n

From the above algorithm 2.2, the triplicate graph of a twig TG (T_n) is a disconnected graph with 9n + 6 vertices and 12n + 4 edges. To make it as a connected graph, for convenience, we include an edge $v_1 v_1'$ to the edge set E as defined in the above algorithm. Thus the graph so obtained is called an extended triplicate graph of twig graph T_n and is denoted by ETG (T_n). By the construction, it is clear that, the graph ETG (T_n) has 9n + 6 vertices and 12n + 5 edges.

Illustration 2.2:

The extended triplicate graph of twig ETG (T_3) is given in figure 1.

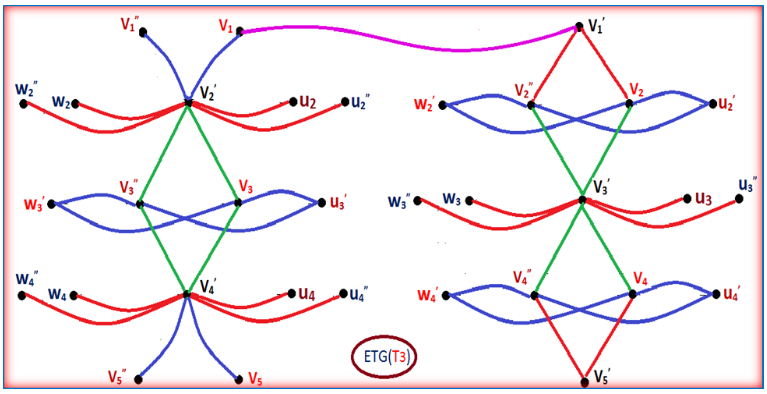

Figure: 1

2.3 Total Cordial Edge Magic Labeling

A graph G (V, E) is said to admit total cordial edge magic labeling if f: V ∪ E → {0, 1} such that

(i) {f (v_i) + f (v_j) + f ($v_i v_j$)} (mod 2) is a constant for all edges $v_i v_j \in$ E.

(ii) The number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

A graph which admitstotal cordial edge magic labeling is called total cordial edge magic graph.

2.4 Total Cordial Vertex Magic Labeling

A graph G (V, E) is said to admit total cordial vertex magiclabeling if f: V ∪ E → {0, 1} such that

(i) {f (v_i) + Σ f ($v_i v_j$)} (mod 2) is a constant for all $v_i \in$ V and the sum is over all vertices v_i adjacent to v_j.

(ii) The number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

A graph which admits total cordial vertex magic labeling is called total cordial vertex magic graph.

TOTAL CORDIAL EDGE MAGIC LABELING

In this section, we present an algorithm to label the vertices and edges of the extended triplicate graph of a twig T_n and prove the existence of the total cordial edge magic labeling for the ETG (T_n)

Algorithm 3.1:
Input *Extended triplicate graph of twig*
Procedure (total cordial edge magic labeling for ETG (T_n))

$V \leftarrow \{v_1, v_2, \ldots, v_{n+2}, u_2, u_3, \ldots, u_{n+1}, w_1, w_2, \ldots w_{n+1}\}$

for i = 1 to n+2 **do**

$v_i \leftarrow 1$

$v_i' \leftarrow \begin{cases} 0, & i \equiv 0 (mod\ 2) \\ 1, & otherwise \end{cases}$

end for

$v_1'' \leftarrow 0$

for i = 2 to n+2 **do**

$v_i'' \leftarrow 1$

end for

for i = 2 to n+1 **do**

$u_i \leftarrow u_i' \leftarrow u_i'' \leftarrow w_i' \leftarrow 1$

$w_i \leftarrow w_i'' \leftarrow \begin{cases} 0, & i \equiv 0 (mod\ 2) \\ 1, & otherwise \end{cases}$

end for

for i = 1 to n + 1

$v_i v_{i+1}' \leftarrow \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$

$v_i' v_{i+1} \leftarrow v_i' v_{i+1}'' \leftarrow \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$

end for

$v_1'' v_2' \leftarrow 0$

for i = 2 to n+1

$v_i'' v_{i+1}' \leftarrow \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$

end for

for i = 2 to n + 1 **do**

$$v_i'u_i \leftarrow v_i'u_i'' \leftarrow \begin{cases} 1, & i \equiv 0 (mod\ 2) \\ 0, & otherwise \end{cases}$$

$$v_i u_i' \leftarrow v_i w_i' \leftarrow v_i' w_i \leftarrow v_i' w_i'' \leftarrow v_i'' u_i' \leftarrow v_i'' w_i' \leftarrow 0$$

end for

$$v_1 v_1' = 0$$

end procedure

Output: *labeled ETG (T_n)*

Theorem 3.1:

ETG (T_n) is a total cordial edge magic graph.

Proof:

We know that, the extended triplicate graph of a twig has 9n + 6 vertices and 12n + 5 edges. Using algorithm 3.1, define a function f: V∪E → {0,1} to label the vertices and edges.

Thus the number of 'zeroes' and 'ones' on the vertices and edges are as follows:

ETG(T_n)	n ≡ 0 (mod 2)		n ≡ 1 (mod 2)	
	1	0	1	0
Vertex label	$\frac{15n+8}{2}$	$\frac{3n+4}{2}$	$\frac{15n+7}{2}$	$\frac{3n+5}{2}$
Edge label	$\frac{6n+2}{2}$	$\frac{18n+8}{2}$	$\frac{6n+4}{2}$	$\frac{18n+6}{2}$
Total	$\frac{21n+10}{2}$	$\frac{21n+12}{2}$	$\frac{21n+11}{2}$	$\frac{21n+11}{2}$

From the table, it is clear that the number of vertices and edges labeled together with '0' and '1' differ by atmost one.

In order to prove the extended triplicate graph of a twigtotal cordial edge magic, define the induced map f*: E → {0,1}such that for any v_iv_j∈ E, f*(v_iv_j) = (f(v_i) + f(v_j) + f(v_iv_j)) (mod 2) = k, a constant.

Thus for all v_iv_j ∈ E, the induced function yields a constant '0'.

Hence ETG (T_n) admits total cordial edge magic labeling.

Example 3.1

ETG (T_3) and its total cordial edge magic labeling is shown in figure 2.

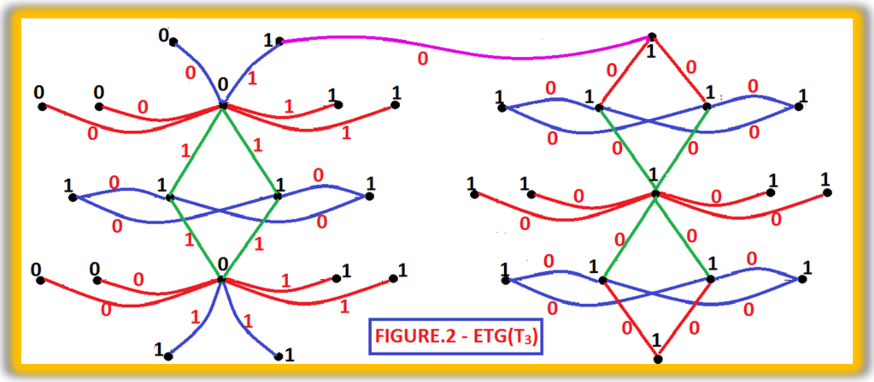

Figure: 2

TOTAL CORDIAL VERTEX MAGIC LABELING

In this section, we present an algorithm to label the vertices and edges of the extended triplicate graph of a twig T_n and prove the existence of the total cordial vertex magic labeling for the ETG (T_n).

Algorithm 4.1

Input Extended triplicate graph of twig

Procedure (total cordial vertex magic labeling for ETG (T_n))

$V \leftarrow \{v_1, v_2, .. v_{n+2}, u_2, u_3, \ldots, u_{n+1}, w_1, w_2, \ldots w_{n+1}\}$

$v_1 \leftarrow v_{n+2} \leftarrow v_1' \leftarrow v_{n+2}' \leftarrow 0$

for i = 2 to n+1 **do**

$v_i \leftarrow 1$

$v_i' \leftarrow \begin{cases} 0, & i \equiv 0 (mod\ 2) \\ 1, & otherwise \end{cases}$

end for

for i = 1 to n+2 **do**

$v_i'' \leftarrow 1$

end for

for i = 2 to n+1 **do**

$u_i \leftarrow 1$

$u_i'' \leftarrow \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$

$u_i' \leftarrow w_i \leftarrow w_i' \leftarrow w_i'' \leftarrow 0$

end for

for i= 1 to n + 1**do**

$v_i'' v_{i+1}' \leftarrow v_i' v_{i+1}'' \leftarrow 1$

$v_i v_{i+1}' = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$

$v_i' v_{i+1} = \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$

end for

for i = 2 to n + 1 **do**

$v_i' u_i'' \leftarrow \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$

$v_i' u_i \leftarrow v_i'' w_i' \leftarrow v_i w_i' \leftarrow 1$

$v_i' w_i \leftarrow v_i' w_i'' \leftarrow v_i u_i' \leftarrow v_i'' u_i' \leftarrow 0$

end for

$v_1 v_1' = 0$

end procedure

Output: *labeled ETG (T_n)*

Theorem 4.1:

ETG (T_n) is a total cordial vertex magic graph.

Proof:

We know that, the extended triplicate graph of a twig has 9n + 6 vertices and 12n + 5 edges. Using algorithm 4.1, define a function f: V∪E → {0,1} to label the vertices and edges.

Thus the number of 'zeroes' and 'ones' on the vertices and edges are as follows:

ETG(T_n)	n ≡ 0 (mod 2)		n ≡ 1 (mod 2)	
	1	0	1	0
Vertex label	$4n+2$	$5n+4$	$\frac{8n+4}{2}$	$\frac{10n+8}{2}$
Edge label	$\frac{13n+6}{2}$	$\frac{11n+4}{2}$	$\frac{13n+7}{2}$	$\frac{11n+3}{2}$
Total	$\frac{21n+10}{2}$	$\frac{21n+12}{2}$	$\frac{21n+11}{2}$	$\frac{21n+11}{2}$

From the table, it is clear that the number of vertices and edges labeled together with '0' and '1' differ by atmost one.

In order to prove the extended triplicate graph of a twigtotal cordial vertex magic, define the induced map f^*: V→ {0,1}such that for any $v_iv_j \in E$, $f^*(v_i) = (f(v_i) + \Sigma f(v_iv_j)) \pmod 2) = k$, for all $v_i \in V$ and the sum is over all vertices v_i adjacent to v_j.

Thus for all $v_i \in V$, the induced function yields a constant '0'.

Hence ETG (T_n) admits total cordial vertex magic labeling.

Example 3.1

ETG (T_3) and its total cordial vertex magic labeling is shown in figure 3

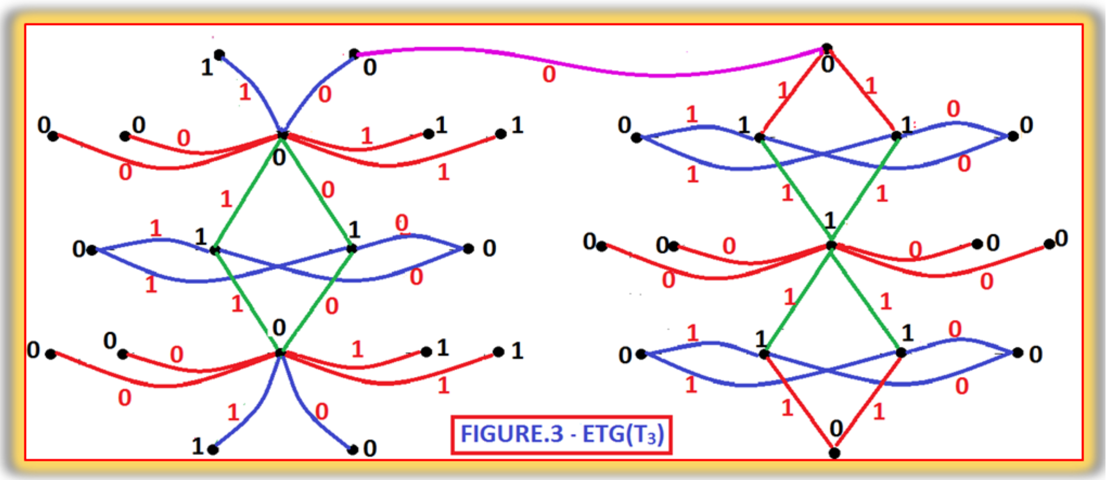

Figure: 3

CONCLUSION

In this paper, we investigated the existence of total cordial edge magic labeling and total cordial vertex magic labeling for the extended triplicate graph of twig by presenting algorithms and with some illustrations.

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